

Time-frequency conversion, temporal filtering, and temporal imaging using graded-index time lenses

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We propose several applications of graded-index (GRIN) time lenses including time–frequency conversion (time-to-frequency conversion and frequency-to-time conversion simultaneously), temporal filtering, and temporal imaging. The evolution of the signal pulses in these systems is demonstrated. As two important parameters, the focal length and the time–frequency conversion factors of time–frequency conversion based on GRIN time lenses are evaluated. © 2012 Optical Society of America

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Using a space–time duality that originates from the equivalence between the paraxial diffraction of a spatial field and the dispersive propagation of a temporal field, temporal imaging [1], time-to-frequency conversion [2], and frequency-to-time conversion [3] can be realized in analogy with spatial imaging systems. These systems have many applications on ultrafast optical processing and measurement. In these systems, the key element is a time lens that is a temporal counterpart of a space thin lens. A graded-index (GRIN) space lens, which can be seen as a space thick lens, has two important properties, i.e., imaging and light focusing. It can be applied in image-transmission systems and optical communication systems [4]. The corresponding principle of a GRIN time lens can be established from a GRIN space lens [5]. Due to the coexistence of dispersion and phase modulation in a GRIN time lens, the method of the GRIN time lens is more exact than the “time thin lens” (traditional time lens) approach [5], which totally neglects dispersion. Therefore, a GRIN time lens has the potential to be used in ultrafast optical processing and measurement as well. To the best of our knowledge, there is still no report on the applications of the GRIN time lens. In this Letter we propose several applications of GRIN time lenses, including time–frequency conversion (time-to-frequency conversion and frequency-to-time conversion simultaneously), temporal filtering, and temporal imaging. Specifically, we demonstrate that when a GRIN time lens operates on an optical signal, the signal can enter a regime where the input signal amplitude is mapped from the time domain into the frequency domain, i.e., time-to-frequency conversion, and at the same time the input signal spectrum is mapped from the frequency domain into the time domain, i.e., frequency-to-time conversion. Furthermore, a temporal filtering system, which is established by two GRIN time lenses and a filter, can realize filtering in the time domain. In addition, a temporal imaging system consisting of a GRIN time lens and a dispersive element can stretch a temporal waveform.

The GRIN time lens can be seen as a “time thick lens,” in which there are dispersion and phase modulation simultaneously. The propagation of optical pulses in a GRIN time lens can be described by [5]

$$i \frac{\partial A(\xi, \tau)}{\partial \xi} + \frac{\beta_2}{2} \frac{\partial^2 A(\xi, \tau)}{\partial \tau^2} + \frac{\phi(\xi, \tau)}{\xi} A(\xi, \tau) = 0, \quad (1)$$

where $A(\xi, \tau)$ is the slowly varying envelope of the pulse, β_2 is the group-velocity dispersion (GVD) value, and $\phi(\xi, \tau)$ represents the quadratic phase modulation in the GRIN time lens. The GRIN time lens can impose a phase $\phi(\xi, \tau) = \Gamma_0 \omega_m^2 \tau^2 / 2$ on the pulse, in which ω_m is the modulation angular frequency and Γ_0 is the phase modulation amplitude of the GRIN time lens.

We propose preceding the GRIN time lens in a temporal Fourier transform (time–frequency conversion) system as shown in Fig. 1. This configuration can convert the temporal (spectral) profile of the input to spectral (temporal) profile of the output, which is similar to a temporal 2-f system (2-f represents including two focal lengths) consisting of three cascading parts, i.e., dispersion, phase modulation, and dispersion [6,7]. For a GRIN time lens with any β_2 and Γ_0/ξ , the time–frequency conversion can be realized at a certain length, which can be called a focal length. Although the implementation of this configuration needs a GRIN time lens with an exact length, i.e., focal length, this configuration can still offer some simplicity, as it avoids the accurate balance of dispersion and phase modulation in the temporal 2-f system, which is two matched dispersions on both sides of the time lens [6].

For an arbitrary input pulse envelope expressed in terms of the Hermite–Gaussian functions [5],

$$A(0, \tau) = \sum_{l=0}^{\infty} a_l \varphi_l(\tau), \quad (2)$$

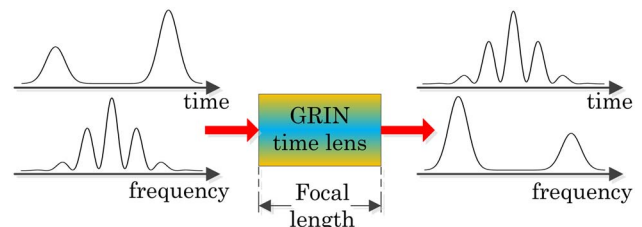


Fig. 1. (Color online) Temporal Fourier transform (time–frequency conversion) configuration.

in which

$$a_l = \int_{-\infty}^{+\infty} A(0, \tau) \varphi_l(\tau) d\tau, \quad (3)$$

$$\varphi_l(\tau) = (2^l l! \sqrt{\pi})^{-1/2} H_l(\tau/\sigma_\tau) \exp(-\tau^2/2\sigma_\tau^2). \quad (4)$$

H_l is the Hermite polynomial of order l , $\sigma_\tau = (\beta_2 \xi / \Gamma_0 \omega_m^2)^{1/4}$ is a scale parameter, and the pulse envelope at an arbitrary position ξ in the GRIN time lens becomes [5]

$$A(\xi, \tau) = \exp(i\Gamma_0 \xi) \sum_{l=0}^{\infty} a_l \varphi_l(\tau) \times \exp[-i(\omega_m^2 \Gamma_0 \beta_2 / \xi)^{1/2} (l + 1/2) \xi]. \quad (5)$$

The Fourier transform of the input pulse envelope is [8]

$$F[A(0, \tau)] = \sum_{l=0}^{\infty} a_l \varphi_l(\sigma_\tau^2 \omega) \sqrt{2\pi} \sigma_\tau \exp(-il\pi/2). \quad (6)$$

Comparing Eqs. (5) and (6), we observe that the focal length L is

$$L = -\pi^2 / (4\omega_m^2 \Gamma_0 \beta_2). \quad (7)$$

Interestingly, the focal length of a GRIN time lens is similar to that of a traditional time lens, which is [2]

$$L' = -1 / (\omega_m'^2 \Gamma_0' \beta_2'), \quad (8)$$

in which ω_m' and Γ_0' are the modulation angular frequency and phase modulation amplitude of the traditional time lens, respectively, and β_2' represents the GVD value of the dispersion.

We simulate the temporal Fourier transform system with various input waveforms. Each input waveform entering the GRIN time lens is Fourier transformed at the focal length. For example, Fig. 2 shows the evolution of the temporal shape and the spectrum of the signal pulses for the case $\beta_2 = 27.9 \text{ ps}^2/\text{km}$, $\omega_m = 2\pi \times 10^{10} \text{ rad/s}$, and $\Gamma_0 = 4\pi$. Obviously, at the focal length of 1783 m, the temporal shape and the spectrum of the output are accurately similar to the spectrum and the temporal shape of the input, respectively. According to Eq. (1), we calculate the relationship of the phase modulation amplitude and focal length by observing the simulation results of a GRIN time lens, which is seen as a process

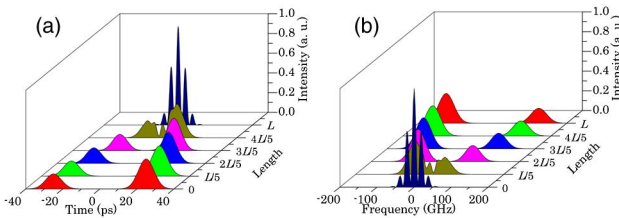


Fig. 2. (Color online) Evolution of the signal pulses along the GRIN time lens in which L is the focal length. (a) Temporal shape evolution and (b) spectrum evolution.

including dispersion and quadratic phase modulation at the same time. In the simulation, the parameters β_2 and ω_m are $27.9 \text{ ps}^2/\text{km}$ and $2\pi \times 10^{10} \text{ rad/s}$, respectively. The phase modulation amplitude versus focal length calculated in the simulation (circles) is depicted in Fig. 3, which is in accord with that analyzed according to Eq. (7) (solid curve), and this accordance can be seen as evidence for the validity of Eq. (7).

The time–frequency conversion factor is an important parameter for a time–frequency conversion system. By comparing Eqs. (5) and (6), this scaling factor can be obtained as

$$\Delta t / \Delta \omega = -2\beta_2 L / \pi. \quad (9)$$

This scaling factor is similar to that of a traditional time lens, which is [6]

$$\Delta t / \Delta \omega = -\beta_2' L', \quad (10)$$

in which L' is the focal length of the traditional time lens. In Eqs. (9) and (10), Δt is the temporal shift of the input signal and $\Delta \omega$ is the resulting spectral shift. This result can be proved by the coincidence of the output and the scaled input according to Eq. (9) as shown in Fig. 4.

To implement a GRIN time lens, a method based on a phase-modulated electro-optic crystal was proposed in [5]. Typically, the phase shift of an electro-optic phase modulator is confined to several π [6], and therefore a large dispersion is needed according to Eq. (7). By enhancing the GVD value, the focal length may be reduced to a practical level. Another promising way is using cross-phase modulation and dispersion in a highly nonlinear fiber (HNLF). The propagation of optical pulses in the HNLF is described by a nonlinear Schrödinger equation,

$$i \frac{\partial A}{\partial \xi} + \frac{\beta_2}{2} \frac{\partial^2 A}{\partial \tau^2} + \gamma |A_p|^2 A = 0, \quad (11)$$

which is similar to Eq. (1). In fact, the maintenance of the pump pulse shape $|A_p(\xi, \tau)|^2$ is the key work for this implementation method.

Besides time–frequency conversion, a basic Fourier optics image processor called the 4-f system, which is established by two temporal 2-f systems and a filter, can realize temporal filtering [7]. By using the similarity of the time–frequency conversion system based on the GRIN time lens and the temporal 2-f system, a new

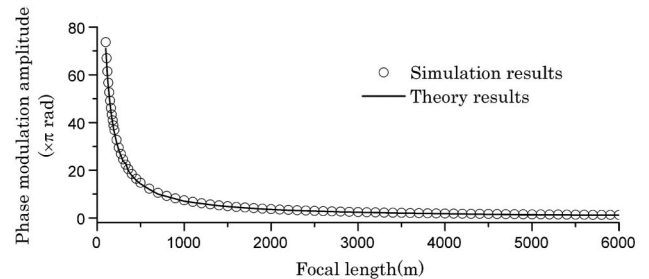


Fig. 3. Phase modulation amplitudes calculated in the simulation (circles) and those analyzed according to Eq. (6) (solid curve) versus focal length.

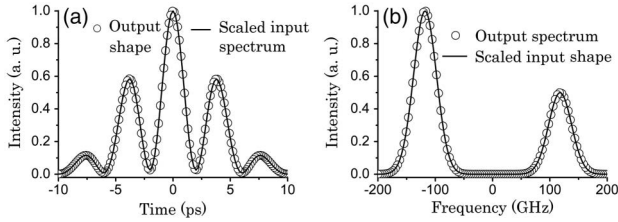


Fig. 4. Output (circles) at the focal length of a GRIN time lens and scaled input (solid curve) according to Eq. (9). (a) The output temporal shape and scaled input spectrum and (b) the output spectrum and scaled input temporal shape.

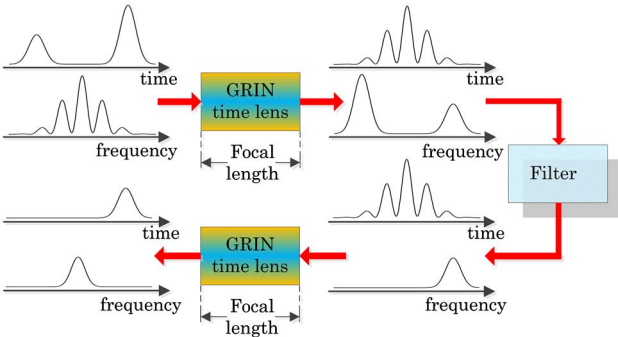


Fig. 5. (Color online) Temporal filtering configuration.

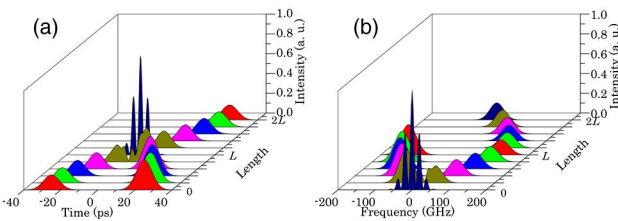


Fig. 6. (Color online) Evolution of the signal pulses along the temporal filtering system. (a) Temporal shape evolution and (b) spectrum evolution.

configuration of temporal filtering system that consists of two GRIN time lenses and a filter is proposed as shown in Fig. 5. We can clearly observe the temporal filtering process from Fig. 5. The numerical results in Fig. 6 demonstrate the evolution of the temporal shape and the spectrum of the signal pulses in the proposed temporal filtering system. By filtering the signal in the frequency domain, we can obtain the corresponding time filtered signal. With the similar method, temporal imaging, temporal correlation and convolution, and a joint transform processor based on GRIN time lenses can be realized in analogy with the existing theories in the 4-f system [7].

Temporal imaging (or time stretching) is another important application of a traditional time lens. Since a dispersive element can realize the frequency-to-time conversion according to the spectral Fraunhofer regime [9], we set up a temporal imaging system with a time-to-frequency conversion subsystem based on a GRIN time lens and a frequency-to-time conversion subsystem based on a dispersive element, which is shown in Fig. 7. By multiplying the conversion factors of two subsystems [9], the magnification factor can be readily obtained as

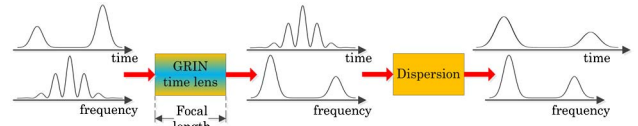


Fig. 7. (Color online) Temporal imaging configuration.

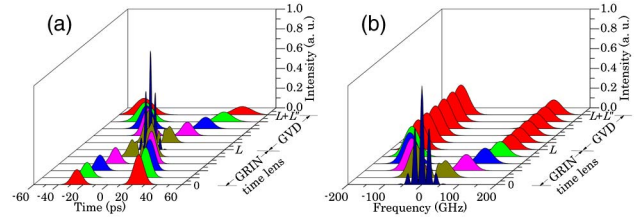


Fig. 8. (Color online) Evolution of the signal pulses along the temporal imaging system. (a) Temporal shape evolution and (b) spectrum evolution.

$$M = -\pi\beta_2''L''/(2\beta_2L), \quad (12)$$

in which β_2'' and L'' are the GVD value and the length of the dispersive element, respectively. The evolution of the temporal shape and the spectrum of the signal pulses in the proposed temporal system are shown in Fig. 8, where $\beta_2''L'' = \beta_2L$. Both the temporal shape and the spectrum of the output are reversed scaled replicas of the input temporal shape, and the signal pulses are effectively stretched in the time domain. From Fig. 8 the magnification factor is observed as $\pi/2$, which is exactly in accord with the theory value calculated according to Eq. (12).

In conclusion, we have described three applications of a GRIN time lens including time–frequency conversion, temporal filtering, and temporal imaging. These applications have the potential to be used in ultrafast optical processing and measurement in the future.

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